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The Discovery of Quantum Mechanics

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CHAPTER TWO

THE DISCOVERY OF QUANTUM MECHANICS

SECTION I: WERNER HEISENBERG AND QUANTUM MECHANICS [1]

Introduction

The insight which led Heisenberg in 1925 to the formulation of quantum mechanics was in some respects as momentous as the Copernican insight into the ordering of the heavenly bodies; for it changed the point of perspective from which physicists since the time of Copernicus were accustomed to look at the world. It changed a viewpoint about the world which had become *classical* and tumbled down a pile of certainties on which the physics' three hundred years had been based. Heisenberg called these the ‘‘ontology of materialism", that is, the certainty that nature was out there, solid and material, infinitely accessible to objective description, in which the goal of each succeeding generation of scientists was the conguering of yet another decimal place [2]. Quantum mechanics showed that this goal was a mirage; it revealed the presence of a subtle subjectivity at the very heart of the scientific enterprise, and, by so robbing the mind of its solid support, left it as Heisenberg said, "suspended as over an unfathomable abyss" – the unfathomable and mysterious abyss of its own subjectivity [3]. Even in the moment of its conception, Heisenberg, Bohr and the small circle of intimates who surrounded them, knew that the structure of quantum mechanics was of critical importance for more than scientific method. They realized that it destroyed one ontology of nature and profoundly affected the science of the intimate structure of the human mind.

[1] We intend to use the terms "quantunl mechanics" or "matrix mechanics" for Heisenberg's theory of 1925; "wave mechanics" for Schrödinger's theory of 1926, and "quantum theory" as a term of general meaning applicable to both.

[2] W. Heisenberg, *The Physicist's Conception of Nature*, (London: 1958) p. 14.

[3] W. Heisenberg, *Philosophic Problems of Nuclear Science*, (London: 1952), p. 117.

Quantum mechanics

It is our intention to use Werner Heisenberg as our guide to the philosophical world of quantum physics, since he was both one of its founders and one of its most profound interpreters. He was one of the many who, in the decade of 1920–1930, were busy with the problem of trying to reconcile quantum phenomena with the traditional physics of Newton, Maxwell and Laplace. Traditional physics was a very proud and impressive scientific structure. It was endowed at that time with an authority derived chiefly from its logical splendour, which made it a norm not merely for all science, but for all rational thinking. Traditional physics was not just a particular view of physics which might itself be subject to revision. It was *classical physics.* It was, therefore, with an experience like that of a conversion, that physicists found themselves turning inward to examine critically the revered foundations of what they and their colleagues had believed in for three hundred years.

Many of the original founders of the quantum theory have told us about the transition that was then taking place in physics. Some accounts date from the early days of hectic and almost evangelical enthusiasm; others were written in retrospect and in a calmer mood. But all conveyed the conviction that as a result of the discoveries of that decade man had reached a new level of consciousness about the world, himself and the horizon of human knowing [1].

The first successful synthesis of quantum with classical physics was made by Heisenberg in the summer of 1925 [2]. His ideas were taken up immediately by Born, Jordan and Dirac who helped to bring them to

1 The principal accounts of the events of this period recounted by Bohr and Heisenberg are: N. Bohr, "Die Entstehung der Quantenmechanik", in *Werner Heisenberg und die Physik unserer Zeit* (Braunschweig: 1961), IX-XII; and "Discussions with Einstein on Epistemological Problems in Atomic Physics", in *Albert Einstein: Philosopher-Scientist* (New York: Library of Living Philosophers, 1949), 199-242; W. Heisenberg, "Quantenmechanik" Nobel Prize address, in *Les Prix Nobel en I933* (Stockholm: 1935); "Fünfzig Jahre Quantentheorie", *Naturwissen.*, XXXVIII (1951), 49-55; "Erinnerungen an die Zeit der Entwicklung der Quantenmechanik" in *Theoretical Physics in the Twentieth Century,* a Memorial Volume to Wolfgang Pauli, ed. by M. Fierz and V. F. Weisskopf (New York: Interscience, 1960).

2 Werner Carl Heisenberg was born in Würzburg on the 5th of December, 1901. He studied physics at Munich under Sommerfeld, Wien, Pringsheim and Rosenthal, entering the university in 1920. During the winter term of 1922-23, he studied under Born, Frank and Hilbert in Gottingen. He obtained his Ph. D. at Munich in 1923, and his *venia legendi (Habilitation)* at Gottingen in the following year. In the winter of 1924-25, he was Rockefeller Scholar under Bohr at Copenhagen. In 1926 he was appointed lecturer in theoretical physics at the University of Copenhagen. In 1927 he became Prof. Ord. of theoretical Physics at the University of Leipzig. He was awarded the Nobel Prize in 1933. He became Director of the Kaiser Wilhelm Institute, Berlin in 1941, and Prof. Ord. at the University of Berlin. In 1946 he helped to found the Max Planck Institute for Physics in Gottingen. He is now Director of the Max-Planck Institut fiir Physik und Astrophysik, Munich.

near logical completion within a year. Schrödinger, working on the ideas of de Broglie, published his celebrated theory of wave mechanics in the spring of 1926, followed soon by a proof of the equivalence of his theory with that of Heisenberg. Within a year, the permanent lines of a new physics were drawn.

The most detailed and authoritative account of the germination of the ideas which constituted quantum mechanics was written by Heisenberg himself for the memorial volume, *Theoretical Physics in the Twentieth Century,* and dedicated to the memory of Wolfgang Pauli [1]. It was written while Pauli was still alive but, by the time of its publication in 1960, Pauli was already dead. In this detailed account of the course and development of his thought in those days, full of personal reminiscences and documented by extracts from his letters of that period, Heisenberg singles out Pauli as his principal confidant and correspondent in the dialogue preceding the fruition which took place in his mind in the summer of 1925.

The questions which were in the air at that time among physicists were three: the anomalous Zeeman effect due to electron spin, the Exclusion Principle, and the foundations of what is now called, the old quantum theory. This was the quantum theory of Bohr and the wave-particle dualism of de Broglie. It was generally thought then that these three questions were connected parts of one problem. As it turned out, however, they were separate questions, each contributing in its way to the overthrow of the scientific outlook of classical physics [2].

As we are principally interested in the change in intentionality marking a shift in the noetic orientation of the physicist-Heisenberg, we shall start at the logical *terminus a quo,* namely, the intentention-ality-structure characteristic of the classical physics.

SECTION II: INTENTIONALITY STRUCTURE OF CLASSICAL PHYSICS

Classical physics is characterised by a naively realist outlook (called "materialist" by Heisenberg) towards physical reality. The physical reality envisaged by the intentionality-structure of classical physics is one made up of the kind of parts which are objectifiable in Space

1 W. Heisenberg, *Erinnerungen* usw., *loco cit.*

2 Cf., for example, Sir Edmund Whittaker, *History of the Theories of Aether and Electricity, 1900-1926* (London: Nelson, 1953).

and Tinle. The outlook of classical physics then implies certain philosophical doctrines about (a) objectivity, (b) causality and (c) reality.

(a) The physical object has *empirical objectivity.* It is a *Gegenstand,* situated out there, *vis-à-vis* the observer [1]. The relationship between noema and noesis is one of exteriority with respect to the knowing subject. For most classical physicists, the physical real is a body situated outside them and outside all observers as such in a determinate part of space and time. It possesses that kind of empirical objectivity we called bodily objectivity. It is made up of parts which, no matter how small they may be, can be represented in a determinate fashion in space. It is composed then of parts which are in turn composed of smaller parts until the smallest parts - if there are such - disappear below the threshold of measurement, observatiop or empirical intuition. However, since the parts at this stage, even though no longer capable of being given in perception, can still be thought about, they are *ideal bodies,* the content of a concept constructed as a limiting case of what is given in experience. There are two such linliting cases: a *classical particle* and a *classical field.* The former has position but no magnitude; the latter is conceived to be an infinitely extended medium like a hypostatised space with just sufficient "body" to sustain vibratory motions.

It should be noted that Kant – the great philosopher of classical physics – was unwilling to allow the scientific object more than phenomenal objectivity since he believed that the realm of the thing-in-itself was unattainable by natural science. The influence of Kant's transcendental critique was not generally felt by the majority of physicists; its effect, however, in the period of crisis which was to accompany the discovery of quantum mechanics was profound.

(b) Physical objects are linked by the kind of causality which regulates their appearances in strict and orderly sequences of antecedent-consequent. For most physicists this causality was between real bodies and could be called *bodily causality.* The follower of Kant would see in it no more than *phenomenal causality.* The complete expression of this point of view is the *physical law of causality,* which is expressed as follows: "When all determinations which describe the present state of an isolated system are known, then the future of the system can be calculated" [2].

[1] Cf. A. Dondeyne, *La différence ontologique chez M. Heidegger* (Louvain, Inst. Sup. de Phil.) p. 11.

[2] W. Heisenberg, "Kausalgesetz und Quantenmechanik", *Erkenntnis,* II (1931), pp. 172-182; quotation is on p. 174.

(c) The physical object has the *public objectivity of a concept*, i.e. it is one which is represented conceptually in the same way by everyone. It has, then a determinate description or definition which leaves no element to be completed by private acts of observation. Public objectivity in this sense is also in the classical world-view a criterion of physical reality: it is, accordingly, a rationalism in which the meaning of "reality" is the content of an infinitely precise conceptual definition from which is excluded whatever is represented by the vague and imprecise elements recorded by concrete empirical intuition; "reality" means "what can be precisely defined even to an infinity of decimal places". This almost Platonist notion of reality dehumanised and taken out of its context in a World of real beings is what Heidegger called *Vorhandenheit* [1]. This is itself one of the extremes in the dialectic of being in Western philosophy; it is the end of one swing of thought and the beginning of another which was to be set in motion by the discovery of the quantum theory; for the first immediate effect of the quantum theory was to reinstate the immediate object of empirical intuition in the centre of science and to focus attention on the material, individual, incommunicable and concrete object of experience as part – and, to many as the whole – of the true object of scientific knowledge.

The classical notion of what constituted a real physical thing and object of physics was founded upon a Cartesian Mind-Body Parallelism in which Mind was thought to "reflect" Matter as in a "mirror" [2]. The classical scientist, then, got to know reality by making infinitely precise this image within him. All that was obscure, indeterminate or indistinct was eliminated as coming from the subject; secondary qualities like colour, taste, etc., were excluded by this criterion. Only the primary qualities of extension and its derivatives were accepted as objective elements of reality, and these only in so far as they were idealised through the assignment of infinitely exact numerical values, which were accepted as belonging to the thing in itself and not to the representation of the thing. The fund of possible physical realities, then, was made up of whatever could be represented by *idealised imaginative models.* These were limiting cases of phenomenal objects to which corresponded the three divisions of classical realities: *classical particles, structures made up of classical particles* and *classical fields.* In summary, then, the

1 Dondeyne, *op. cit.*, p. 20, where the author refers to paragraphs 19, 20 and 22 of Heidegger's *Sein und Zeit.*

2 W. Heisenberg, "The Origins of the Mechanistic and Materialistic World-View" and "The Crisis of the Mechanistic-Materialistic Conception", *Physicist's Conception ot Nature,* pp. 121-179.

classical physicist oriented himself to the construction of idealised and objectifiable phenomenal objects, i.e. concretely, to an explanation in terms of classical particles, spatially constructed models and classical fields [1]. We have called this the intentionality-structure of classical physics.

Out of this account the main theme of our study arises. This is an analysis of the various kinds of objectivity in modern quantum physics with a view to separating the scientific object from the forms imposed upon it by human knowing, and with a view to studying critically the possible link between the scientific object and reality.

SECTION III: CRISIS OF THE CLASSICAL INTENTIONALITY-STRUCTURE

Crisis

The three problematic areas of quantum physics, viz, spin, the exclusion principle and the failure of the old quantum theory, could not be reconciled with the picture of reality given by classical physics. Spin was a mysterious new dimension. The exclusion principle forbade for no clear reason the duplication of like bodies. The old quantum theory, while satisfying the classical criteria of objectivity, nevertheless allowed the electron within the atom to violate well-established classicallaws. Moreover, it was found that the old quantum theory which gave good results when applied to the hydrogon atom, failed in most other cases and notably when applied to the hydrogen molecule.

Heisenberg, recounting with scrupulous care the source of his ideas, says that in October, 1923, Pauli was the source for him of a great light on the meaning of physics: model representations, Pauli said, had in principle "only a symbolic sense", they were "classical analogues for a 'discrete' quantum theory" [2]. The remark was momentous, not because it attacked any physical result, but because it attacked the intentionality-structure which supported classical physics and which hitherto was accepted as the only reasonable dynamic structure capable of generating a valid physical theory. The consequences of this

1 Although the classical object was conceived to be something in the three-dimensional space of our experience, it was not an object of perception in the rich emotive personal way of everyday life. It was already a very abstract construction. It was because of this that Goethe and the humanists of this century and the last have cried out against the claims of physical science to represent reality truly. Cf., Heisenberg, *Philosophic Problems* etc., pp. 60-76, and C. F. von Weizsäcker, *The World View of Nature,* pp. 93-94.

2 Heisenberg, *Erinnerungen* usw., *loco cit.*

change in viewpoint were profound. If the phenomenal object is only a symbol of reality, then reality is what lies "behind" the symbol and may possibly be unknowable. The swing away from rationalism had begun. It opened the door to the two extremes between which philosophers of physics have since been divided: *Empiricism* or *Empiricistic Positivism* on the one hand, which denies the possibility of an ontology of nature, and *Subjectivism* on the other hand, which sought the meaning of reality in evolving noetic experience alone, apart from a transcendent reality revealed through it. Heisenberg certainly rejected the former, and Pauli with him in all probability. The philosophy to which Heisenberg eventually settled down was a Kantian-style Idealism in which a tenuous thread linked the noetic experience to an unknowable noumenal term.

A Physics of "Observables"

The great insight which brought about the discovery of quantum mechanics was that physics should concern itself only with *observable quantities.* The insight came to him in May, 1925, as he was about to leave for a vacation in Heligoland. During the month of June on Heligoland he sketched the application of his idea to the anharmonic oscillator and found that it worked. This was the subject of his first paper on quantum mechanics, submitted to the *Zeitschrift für Physik,* and was received by the editor on 29 July 1925 [1].

The content of that insight was remarkable; not merely because it inaugurated a new era in physics and a new intentionality-structure in science, but because, important as it was, its precise content eludes definition. It has an air of deceptive simplicity. At first sight, it has all the appearance of a refreshingly clear, matter of fact, down-to-earth statement which delights the practical man by cutting away the myth and mystification of an entrenched tradition. And it was in this sense that it inspired a kind of iconoclastic uprising among the young, positivistically inclined physicists whose evangelical motto became "Out with metaphysics and all unobservable quantities!". A closer inspection however shows that Heisenberg's basic insight was one of Teutonic complexity of whose meaning and implications Heisenberg himself was not fully aware. We shall try to bring out some of these implications and use them to throw light on the main problem of this thesis.

[1] W. Heisenberg, "Ueber quantentheoretische Umdeutung kinematischer und mechanischer Beziehungen", *Zeit. f. Physik,* XXXIII (1925), pp. 879-893.

Heisenberg wrote to Pauli in a letter of 24 June 1925 about his master-idea: "Grundsatz ist bei der Berechnung von irgendwelchen Grossen, wie Energie, Frequenz usw., dürfen wir nur Beziehungen zwischen prinzipiell beobachtbaren Grossen vorkommen" [1]. The basic principle, he says, is to consider only relations between observable magnitudes, that is, between magnitudes which could in theory be observed.

But what is an observable? Taking the term in an unqualified sense, an observable is whatever can manifest itself immediately in experience, like heat (as felt), colour (as seen), sound (as heard), etc. At first sight, this seems to be what Heisenberg means when he criticises intra-atomic electron orbits as "lacking intuitive foundation" [2]. However, it was not Heisenberg's wish to deny the three hundred years of physics based upon the mathematisation of qualities *as measured* in order to return to a pre-Galilean or Aristotelian physics based upon qualities *as sensed.* What stimulated Heisenberg's insight was the recognition that certain variables, like the intra-atomic electron orbits, appearing in the old quantum theory, were *not measurable.* They were, in fact, not even imaginable, for the imagination cannot picture radii of 10^{-8} cm. The electron orbits were limiting cases of the imaginable and so were concepts. But in so far as imaginative representations are used, these were merely imaginative *symbols* of something that escaped the power both of imagination and of measurement. Was it, however, the absence of a true image of them or the failure of measurement technique-for the electron lacked both-which made them unobservable? We argue that it was not the mere absence of a true image; for Heisenberg continued to speak of the "observation of electrons in an atom" [3]. Many physical properties, like magnetic field, the polarization of light, etc., produce no specifically recognisable effect directly on the senses or imagination; they have no true image. Their essence is in the way they influence other things and it is not important that they should be capable of being experienced directly. We conclude then that *observable* and *unobservable* are to be defined with reference to *measurability.*

Measurability, however, is a complex notion. It involves an interaction with a measuring instrument capable of yielding macroscopic sensible data, and a theory capable of explaining what it is that is

1 Heisenberg, *Erinnerungen* usw., *lac. cit.*

2 Heisenberg, *Zeit f. Physik*. XXXIII (1925), p. 879.

3 W. Heisenberg, *The Physical Principles of the Quantum Theory.* (Chicago: Univ. of Chicago, 1930), p. 64; the same point is also implied in the article we are considering.

measured and why the sensible data are observable symbols of it. Heisenberg's notion of observability involves all these points implicitly. The explicit dominant factor in his mind was the necessity of giving a physical quantity an "intuitive foundation" in the measuring process.

In what sense do the sensible data give an "intuitive foundaiton" to the measured quantity? Sensible data are, as we have said, *observable symbols* of the property. However, to *observe something* is in principle different from *observing its symbol.* They are distinct actions and could conceivably exclude one another. To see the word "Dublin" is not to observe Dublin, even though the word "Dublin" is the symbol of Dublin. Is then the observable of physics merely the observable symbol, or is it a real property revealed in some non-metaphorical way through the observable symbol? One of the aims of this thesis is to study the various answers given by physicists to this question. Our answer is that the observable symbol can reveal a real property if it denotes or indicates the real presence of a variable whose intimate nature, though not *per se* representable in sensibility, is known, however, in some other way and simultaneously. We take the observable symbol to be the criterion of reality for something whose nature is known only as part of a complex relational totality expressed symbolically in linguistic or mathematical terms. The something *beyond* the symbol to which it refers may be a constructed object merely immanent to the knower, or the symbolism may go further and denote a transcendent thing or property. It will be our task to establish criteria for distinguishing these two cases [1]. We call both of them "*observation in the symbol*", and complex though the description is, the kind of process we have described is performed continuously and with ease in daily life; for the use of language is nothing more than to "observe in the word-symbol" something beyond itself, namely, its immanent sense or its (transcendent) referent.

The other important element in Heisenberg's insight was the need he saw to return to the concrete, immediate instance of a physical property as revealed in the data of individual measurements. This involved a turning away from the rationalism of classical physics with its criterion of the clear, distinct and abstract idea, and a rediscovery of reality in the individual, factual instances revealed and mediated by the act of observation. It was on account of this strong empirical

[1] For example, in a language the *semantic* or ***formal*** meaning of a word is a term or object purely immanent to the knowing subject; but its *full* or *ontological* meaning generally refers to a reality transcending the immanent term.

element – a break with three hundred years of physical tradition – that quantum mechanics marched on to the stage accompanied by a militant philosophy of Positivism and Empiricism. However, that was by and large contrary to the inclinations of Heisenberg, who remained attached to the old criterion in philosophy and sought a rationalistic explanation on a deeper level for the indeterminacy and impreciseness of the new physical object. He found it, as we shall show, in a transcendental critique of the new scientific knowledge.

SECTION IV: QUANTUM MECHANICS, A NEW KIND OF PHYSICAL THEORY

A Theory of Operators

We shall postpone the inquiry into Heisenberg's ontology and theory of knowledge to a later chapter. For the moment we shall consider only the immanent object symbolised on the one hand by its observable symbol and on the other by its appropriate mathematical sYmbol.

The object called an "observable" was represented in Heisenberg's first paper by a linear algebraic operator, which Born showed had the properties of a matrix [1]. The eigenvalues of this operator gave the set of possible values of the observable [2]. The set of observables were defined theoretically in such a way as to preserve a reasonable continuity between classical and quantum physical theories in limiting cases. This latter condition was Bohr's *Correspondence Principle* which had been used so successfully in the old quantum theory: we shall return to this later on. The principal difference between classical physical theory and quantum mechanical theory was the substitution in quantum mechanics of a linear operator for the numerical variables of classical physics.

The observable as a linear operator gave more information than the corresponding classical variable. *In the first place,* its set of eigenvalues restricted the range of possible numerical values. This range ceased in every case to be a continuous range, but admitted discrete values and discontinuous jumps. Both the continuous range

1 M. Born and P. Jordan, "Zur Quantenmechanik", *Zeit. f. Physik,* XXXIV (1925), pp. 858-888.

2 The linear operator is assumed to be *Hermitian* and *hypermaximal;* the former guarantees that the eigenvalues are real, the latter that it has a soluble eigenvalue equation. Cf., John von Neumann, *The Mathematical Foundations of Quantum Mechanics,* (Princeton: 1955), pp. 153, 169.

and the discrete values were calculable, in principle, from the theory. *Secondly,* the linear operator, as Born and others were immediately to show, gave also a probability-distribution governing the ideal frequency of occurrence of particular values of the observable within a set of independent observations [1]. And *finally,* since the coordinate observable did not commute algebraically with the corresponding momentum observable, their probability distributions – but not their ranges of possible values – were correlated. The derivation of that correlation, called the Indeterminacy (or Uncertainty) Principle, was made by Heisenberg in 1927[2].

Novelty of Quantum Mechanics

Quantum mechanics was a new kind of physical theory. In the first place, it determined the possible range of values of its own variables, which classical theory left – except in exceptional cases – to factual observation. In the second place, it allowed the calculation not merely of the ideal norm (or expectation value) of sets of concrete data, which was the aim of classical deterministic theory, but also the manner of distribution of individual instances about the expectation value. Here was another radically new result. For, while in a classical deterministic theory like Newtonian mechanics, concrete measured data are distributed about means randomly, independently of the other variables and generally according to a Gaussian law (unless there is reason to assume a different error curve), in quantum mechanics on the other hand the distributions are random, but not independent, and their forms depend on the initial boundary conditions as well as on the equation of development (the Schrödinger equation of the system). In classical physics, statistical theories are separated from deterministic theories: the function of the latter being to define by implicit definition the elements and properties of the underlying statistical ensemble [3]. The great originality of quantum mechanics is that it both

[1] M. Born and P. Jordan, *loco cit.* Heisenberg attributes the probability-jnterpretation to Born and Pauli, adding that the idea had also occurred to himself, cf., *Erinnerungen* usw. cf. also P. A. M. Dirac, "Physical Interpretation of Quantum Dynamics", *Proc. Roy. Soc.*, CXIII (1927), pp. 621-641.

[2] W. Heisenberg, "Ueber den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik", *Zeit. f. Physik,* XLIII (1927), pp. 172-198.

[3] Most statistical theories, like statistical thermodynamics, are under some aspects equivalent to deterministic theories; since the new variables (temperature, entropy, etc.) are defined implicitly with respect to a set of interrelated variables. The statistical element enters when these new variables are identified with certain limiting statistical concepts applied to an underlying ensemble. However, the deterministic part in a classical statistical theory does not go so far as to define the elements and properties of the underlying ensemble. F... purpose it has to rely on a distinct theory like Newtonian mechanics.

defines the properties of the elements of an ensemble and predicts their frequency distribution within the ensemble *in one formalism.* This involves a double interpretation of the same formulism as we shall see.

SECTION V: QUANTUM MECHANICS AND WAVE MECHANICS, 1926

Wave Mechanics

A rival to quantum mechanics was published by Schrodinger in the spring of 1926 [1]. It was a new theory, conceived independently of the insights of quantum mechanics and capable of being interpreted in a contrary sense. It was known as Wave Mechanics. It was a very elegant mathematical theory, based physically upon de Broglie's notion of a matter wave associated with every particle and employing in a grand manner that kind of functional analysis developed for electromagnetic theory which was the crowning glory of traditional physics. The new theory immediately fired the imagination of physicists, while Heisenberg's matrix mechanics left them cold. Schrodinger's elegant mathematics was of a kind known to and deeply respected by most physicists: Heisenberg, on the other hand, had been forced to create a new unfamiliar algebra of repelling abstractness. Furthermore, Schrödinger appealed directly to the imaginable qualities of waves, wave packets, of group and phase velocity which were part of the daily currency of classical physics [2]. Compared with the vividness, elegance and pictorial quality of Wave Mechanics, matrix mechanics was, as Schrödinger put it, "von abschreckender ja abstossender Unanschaulichkeit und Abstraktheit" [3]. Bohr straightaway invited Schrödinger to Copenhagen and in the autumn of 1926, Heisenberg and Schrödinger met to discuss their respective viewpoints, with the presence of Bohr as a moderating influence [4].

Heisenberg and Schrödinger

No *rapprochement* occurred between the principals. Heisenberg rejected wave mechanics and Schrodinger rejected quantum mechanics. Heisenberg argued that wave mechanics was incapable of explaining

1 E. Schrödinger, *Ann. d. Physik,* (4) LXXIX (1926), 361; 489; 734; (4) LXXX (1926),437.

2 For example, E. Schrödinger described the electron as a small wave packet circulating around the nucleus of an atom in *Naturwissen.,* XIV (1926), p. 664.

3 Quoted by Heisenberg in *Zeit. f. Physik,* XLIII (1927), p. 195, footnote.

4 N. Bohr, *Werner Heisenberg* usw., p. x.

quantum discontinuities in the microscopic domain. "The more I think of the physical side of Schrödinger's theory", Heisenberg wrote in the summer of 1926, "the more I find it abhorrent (abscheulich). Schrödinger throws all quantum theory overboard, viz., the photo-electric effect, Franck collisions, the Stern-Gerlack effect, etc. Under these conditions, it is not hard to construct a theory" [1]. Schrödinger rejected equally emphatically, Heisenberg's belief in "quantum jumps" and accused quantum mechanics of being repellingly abstract and unrealistic. Bohr, however, who moderated these discussions, came to the conviction that both theories must be correct since both gave correct results, and urged the adoption of a higher viewpoint in which there was room for both. The name he gave to this higher viewpoint was *complementarity.*

Heisenberg, however, remained firm in the conviction that quantum discontinuities occur in Nature and that they are basic and irreducible data. On 6 November, 1926, the editor of the *Zeitschrift für Physik* received a paper from him entitled "Schwangungerscheinungen und Quantenmechanik", in which he tried to justify this position [2]. He concludes the paper: "A continuous interpretation of the quantum mechanical formalism – and thus also of the de Broglie-Schrödinger wave-does not belong to the substance of these relations. Furthermore, the fact of discontinuities is harmoniously contained in the mathematical scheme of quantum mechanics". The phrase "does not belong to the substance of [quantum or of wave mechanics]" means, in the context, that it cannot be established by *observable criteria.* One would find the conclusion a weak one, if one did not share Heisenberg's master-insight into the nature of physics as a science of observables.

Heisenberg was also stung by Schrödinger's criticisms to defend his theory from the *abstossende Unanschaulichkeit und Abstraktheit* of which it had been accused. During the winter of 1926-1927 Heisenberg and Bohr discussed their different philosophical interpretations of quantum mechanics; Bohr wanting to begin from the acceptance of the complete equivalence of wave and particle pictures, Heisenberg holding to his rejection of wave mechanics and its unverifiable implications of continuity in Nature. Although these discussions took place daily and were often protracted into the night, Heisenberg recounts that "real clarity was not reached", for conflicting conceptual values

1 Heisenberg, *Erinnerungen* usw., p. 44.
2 *Zeit. f. Physik,* XL (1927), pp. 501-506.

(Gedankengut) were involved. "We could not find our way in all these 'matters", was his conclusion [1].

Bohr went off to Norway on a skiing holiday in February, 1927, and Heisenberg took the opportunity to elaborate and clarify his own views. These he sent to Pauli who was in substantial agreement with them. Thus originated one of Heisenberg's most celebrated papers, "Ueber den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik". It was received by the editor of the *Zeitschrift für Physik* on 23 March, 1927[2]. When Bohr returned from his holiday and read the manuscript, he still disagreed with Heisenberg's method and starting point. By this time Bohr had elaborated his own interpretation based upon the *Principle of Complementarity*, to which we shall return presently. Heisenberg concluded his account of this period by remarking that, in spite of philosophical differences, the physical consequences of the two interpretations were the same. The note which he added to the manuscript, in deference to Bohr, indicates the possibility of a wave-particle interpretation such as that suggested by Bohr.

SECTION VI: THE INDETERMINACY RELATIONS OF 1927

The Intuitive Meaning of Quantum Mechanics

In the celebrated paper in question, Heisenberg tried to explain what matrix mechanics means to one whose criterion of intelligibility is bound to pictures, images and concrete operations. The dominant idea, as one would expect, is the notion of an observable as dependent on the possibility of measurement. For example: he explains that the concept of place involves a reference to the way position is measured relative to a frame of reference, *"anders hat dieses Wort keinen Sinn"*. Since the position-measurement of a microscopic particle involves the exchange of at least one photon with the measuring instrument, successive position-data for a particle do not lie on a continuous trajectory, but must be represented as it were by a series of separated dots on a graph. These are the observables with which physics starts, and they are *discontinuous*. There is, consequently, no unique rate of change, no unique momentum at a point. There is an average for the short time-interval before the point and a different average generally for the short interval after the point. Hence, exact knowledge of

1 Heisenberg, *Erinnerungen* usw., p. 46, on which this account is based.
2 *Zeit. f. Physik*, XLIII (1927), pp. 172-198.

position excludes exact knowledge of momentum at that point. This kind of explanation Heisenberg calls the *anschauliche Deutung,* i.e. the *intuitive meaning* of quantum mechanics. It consists in a "qualitative" and in a "theoretical" part, as he says. The theoretical part consists in understanding that the theory is non-contradictory; the qualitative part consists in knowing how the data are experimentally obtained.

The Indeterminacy Relations

Assuming that the position coordinate x of an electron has been measured with a certain degree of accuracy yielding a Gaussian wave packet, Heisenberg then derives the celebrated Indeterminacy (or Uncertainty) Relation:

$$Dx.Dp \geq h$$

where Dx is the standard deviation of the statistical distribution of x-measurements; Dp is the standard deviation of the statistical distribution of p-measurements (where p is the momentum in the x-direction), and h is Planck's constant of action.

All of these points were already *implicit* in Heisenberg's first paper. His discussions with Bohr, and especially his passipnate disagreement with Schrodinger's views, forced him to disentangle some of the complex and tangled threads of that notion to which he had given the deceptively simple name of *an observable.* We have already seen that the essential core of meaning of this concept is *measurability*. From the paper we are considering, it becomes clear that, over and above measurability, quantum mechanics is concerned with the properties of *measured concrete data,'* that these include necessarily an *interaction with a measuring instrument:* that this interaction is responsible for the *discontinuities* of the data (the so-called quantum jumps), and hence for the *indeterminacy* of the slope between successive data points.

This account has many surprising aspects. **In** the first place, it is clear that the very same statements can be made of any system, classical or quantum. Successive determinations are always discrete, discontinuous and affected by what are called "instrumental errors". If the classical trajectory is smooth and continuous, it is only because it does not deal directly with concrete data; the smooth curve is a constructed-theoretical norm whose essential property is that concrete data do not diverge from it systematically. It has a definite slope at every point – identified with the classical velocity– only because the curve is an abstraction. Such an ideal path can also be constructed for the quantum mechanical data – it is the plot of expectation values – and coincides in fact with the classical trajectory. This leads to our *first conclusion,* which we have already stated above, that one of the main differences between classical mechanics and quantum mechanics is that the fonner gives *only* the ideal nonn from which concrete data do not systematically diverge; while quantum mechanics gives, in addition, fonnulae for the way the statistical distributions of concrete data are correlated. In other words, *quantum mechanics unites in a single formalism the functions of both a statistical and a deterministic theory.*

The *second* significant difference between classical and quantum nlechanics is in the fonn of the Indetenninacy Relations. Indeterminacy relations can be constructed in the classical case just as in the case of quantum mechanics, by taking the product of the standard deviations Dx and Dp of the relative departures of x and p from their classical nonns. In the classical case, the probability distributions of Dx and Dp are to be taken as independent, and, unless there is good reason, Gaussian in form. Then Dx and Dp are independent, and there is no theoretical lower limit to the product Dx. Dp. Of course, if one were to try to make the concrete data more and more precise, one should have passed outside the domain of validity of classical mechanics long before Dx.Dp has reached the value of h. Quantum mechanics, however, relates the probability distributions of Dx and Dp to one another and establishes that there is a theoretical lower limit of h to the product of their standard deviations.

SECTION VII: THE INDETERMINACY OF THE FACTUAL

Enriching Abstraction

Some idea of the kind of indetenninacy involved in quantum mechanics can be drawn from the preceding account; for if one aspect of quantum mechanics is concerned with concrete data *as such,* then as a corollary there is a certain indeterminacy with regard to the momentary rate of change of the measured variable. It is the *indeterminateness ot tact* that follows from our way of knowing; for our first contact with the concrete case is through the presentation of sensible symbols. Such a contact is not yet a knowledge of a thing or an object but merely of a symbol of it. Comparison with other instances leads to an insight which is an understanding of what these sense presentations may possibly symbolise. This insight we called *enriching abstraction,* since it adds to the concrete particularity of the data as not-yet-understood, the enrichment of an act of understanding expressing an ideal norm which is essentially the addition of a set of relations between things or between their symbols. The individual case is then known as a sample of an ideal norm, in so far as it is a member of an ensemble of individual cases which do not systematically diverge from the norm (i.e. which have only random divergence from the norm). However, the indeterminacy of fact is joined with determinacy of definition; for definition is by concept and in this case it is the ideal norm.

In quantum mechanics the definition is represented by the observable as a linear operator implicitly defined within a consistent theory and linked to experimental processes by operational definitions. It answers the question: how is position, momentum, etc., defined? The non-commutation of position and momentum operators becomes part of a new definition (or re-definition vis-à-vis the classical definition) of these which changes the *meaning* (or sense) of *position* and *momentum* for quantum systems [1]. The indeterminacy of fact, however, answers the question: what is the value of the position and momentum coordinates of this system here and now? The answer is given by referring to the results of actual measurements. The indeterminacy of fact is related to the determinacy of definition, as concrete instance is to conceptual definition. What is new in quantum mechanics is not that indeterminacies of variables like position and momentum exist, but that, being fonnerly thought independent, they are now seen to be related to one another. The measuring process which enters into the definition of one disturbs the measuring-process which enters into the definition of the other. This is the physical significance of the change in *meaning* of "position" and "momentum", accomplished by the quantum mechanical re-definition. Heisenberg then was strictly correct when he said: "Any use of the words 'position' and 'velocity' with an accuracy exceeding that given by [the Indeterminacy Relations] is just as meaningless as the use of words whose sense is not defined" [2].

Ignorance and Nescience

From these considerations there follows our rejection of human ignorance as the basis of probability laws in physics. Human ignorance

[1] This point is stressed by N. R. Hanson in *Patterns of Discovery* (Cambridge: 1961), chap. VI, and in *Concept of the Positron* (Cambridge: 1963), chaps. II-IV. Cf. chap. V, p. 106.

[2] Heisenberg, *Physical Principles* etc., p. 15.

concerns what we could and should know, but do not know. The indeterminacy of the factual, however, which states our inability to increase without limit the number of decimal places in a concrete determination, belongs in the first place to what we *could not* know. Moreover, it is our view that a fully determinate concrete reality is not expressed by an infinity of decimal places. An ideally exact number is a concept and hence performs the function merely of an ideal norm. Finally, we wish to insist that the numbering belongs only to the observable symbol and not directly to the physical property symbolised by it. There may be minds capable of knowing the concrete physical reality in its particularity – perhaps even the human mind in some poetic or mystic mode of operation can reach it – but the particularity would not be expressed through the medium of number sets, it would be a concrete self-revelation of an object in which number possibly has no part. We propose to call our lack of knowledge of concrete factual cases *nescience* instead of ignorance.

Heisenberg does not distinguish between ignorance and nescience, since he, with practically all physicists, shares the view that a concrete case is one which is precisely defined in the sense that all its physical properties possess an infinity of well-determined decimal places. Consequently, the wave packet which describes the probability distribution of the coordinate values is interpreted by him as an expression of the scientist's ignorance of the real physical state of the particle.

Even though it is evident that there is no concrete determination which could not be bettered in some way, we do not agree that the random aspect inescapable from every concrete datum is justly called ignorance. The data on which a particular physical equation is based are neither ideal data nor even the best data – if by "best" one means "with most decimal places" – but merely good data. Good data are data that respect the fact that only a limited number of decimal places are significant in any given physical context and concentrates on these. If a premium is set on the search for more decimal places in every experimental process, a type of unintelligent industry is encouraged which is the stultification of true scientific work. To be called a "master of judicious approximation", as was said of Fermi, does not imply systematic negligence but, on the contrary, excellence of judgement. We do not mean to deny the value of more and more accurate *all-round* experimental measurements, nor do we underestimate the value of more decimal places in the calculation of an ideal norm. What exists, however, is not an ideal norm but a concrete sample in which only a certain number of decimal places are *in fact* significant, and to know how many are significant in fact is a mark of wisdom, and not ignorance[1].

Hence, when Heisenberg states that the dimensions of a wave packet are determined by the subjective conditions of the knower, viz., his ignorance, we reply that, on the contrary, it is determined by the kind of idealization we need to represent the boundary conditions of the experimental context, namely, of the concrete situation. The wave packet is our way of expressing (i) the circunlstances under which the system was prepared and (ii) the objective probabilities, viz., the ideal frequencies that arise when subsequent measurements are made upon it. It is not the limitations of our knowledge that specify the wave packet, but it is rather the fact that the physical events can no longer be idealised by deterministic correlations in a purely classical way. Initial boundary conditions no longer deternline uniquely (with the appropriate equations) the results of subsequent but otherwise arbitrary measurements that might be made upon the system. A new element has been discovered in the physical situation. Now knowledge has arisen – not on the basis of ignorance as Heisenberg would suggest – but on the basis of a more accurate analysis of the data.

What we have just said points to a certain ;inconsistency between Heisenberg's principle that observables are the matter of physics and his confused view as to what he thinks physics is *really* about. This last is a relic of the rationalism of classical physics which has not been overcome by the new intentionality implicit in his quantum mechanics of observables.

The Relational Structure of Physical Variables

In describing Heisenberg's view above, we stated that some interaction with a measuring instrument was a necessary consequence of the observability of a physical property. We now ask the question: in Heisenberg's view, is the physical property measured by the observable data *essentially* constituted by the interaction between instrument and object, or is the interaction only an accidental but inescapable means of relating the otherwise imperceptible object to the scientist's experience?

1 Sir Arthur Eddington has rightly said: "By 'observation' we mean *good* observation ... ; 'good' is not here taken to mean 'perfect'. By *good* observation we emphatically do not mean *perfect* observation ... The odd thing is that, having made his perfect arrangements, the perfect observer often fails to accomplish things which to the good observer are quite elementary". *The Philosophy of Physical Science* (Ann Arbor: Univ. of Mich. Press, 1958), pp. 96-97.

If physics is or ought to be concerned only with the way things interact with measuring instruments, then the basic observables of physics are *essentially* constituted as relations between things and things, based upon so many different ways in which things act mutually and reciprocally upon one another. The aim of physics, then, would be to discover interrelated sets of these activities. This would seem to be the logical conclusion of Heisenberg's insight. However, Heisenberg was not able to detach himself sufficiently from the rationalist background of classical physics to draw this conclusion. In failing to do so, he spilled from his sails the guiding breeze of his original inspiration and so never really fully overcame the encircling restrictions of the classical intentionality-structure.

This failure led him to retain a parallelistic theory of knowledge, one different, however, from the naive parallelism characteristic of classical physics. If the balance illustrates the relational view of physical science then the microscope illustrates the parallelistic theory of science. The balance compares an object in one scale with a standard unit or a fraction of a unit in the other. A microscope on the other hand merely enlarges the impression the object makes on the eye. The classical physicist looked for an exact image of what was out there. Heisenberg accepted this description: the instrument is to man, as he said, rather as a part of our organism than as a part of external nature or as the snail's shell is to its occupant [1]. He pointed out, however, that the instrument through which we look disturbs what is out there and that we see, consequently, not what is there but something which is in part at least a product of the act of observation. "When we speak of the picture of nature in the exact science of our age", he wrote, "we do not mean a picture of nature so much as *a picture of our relationships with nature*" [2].

The Wave Packet

In the paper we have been considering, the notion of a *wave packet* does not emerge clearly. On the one hand, Heisenberg says that, since it results in no more than a probability distribution for the position of the system, it is merely a measure of the scientist's knowledge or lack of knowledge of the physical system. Because of this, he sometimes calls the wave packet a *probability amplitude* or *probability wave.* On the other hand, since this "probability wave" was capable of interfering

1 Heisenberg, *Physicist's Conception* etc., p. 18.
2 *Ibid.*, p. 29, the author's italics.

with itself like a light wave, he seems also to consider it as more than a mere mathematical function.

His final conclusion is that the probability formulae of quantum mechanics include a reference not only to the kind of experiment which prepared the state, but also to the *kind of experiment which* is *ultimately envisaged.* By this he means that the development of the wave function does not describe a process occurring independently of observation, but that it represents rather a set of incomplete potentialities which need to be completed by a future act of measurement. He does not discuss here how the probability wave connects past and future states or measurements; this was to be one of the central problems of the new physics. His solution at this stage, in spite of the title of the paper, tends to be abstractly intellectual in keeping with his original insight. The course of our epistemological analysis led us back to the views expressed in this paper. Our own solution was inspired by Heisenberg's original insight and tries to make it consistent with itself and with a satisfactory theory of knowledge.

Summary

In this chapter we discussed how Heisenberg's insight of 1925, that physics should concern itself henceforth only with relations between *observables,* changed the intentionality-structure of physics. This insight led him to the construction of a quantum mechanics of observables. We discussed briefly the significance of his insight, and of his rejection of Schrödinger's wave mechanics; the novelty of quantum mechanics as a physical theory, and the meaning he attributed to its most surprising result, viz., the Indeterminacy Relations. We pointed out that the crisis was a crisis of the rationalism inherent in the outlook of classical physics, and that Heisenberg's insistence on "observable quantities" was a return to the individual and empirical manifestations of reality which as such, to our way of knowing, are penetrated with a certain random quality.