2015

The Uncertainty Relations

Patrick Heelan
Georgetown University, heelanp@georgetown.edu

Patrick Aidan Heelan, *The Observable: Heisenberg's Philosophy of Quantum Mechanics.*
With a foreword by Michel Bitbol. Edited and with a foreword by Babette Babich.
Oxford: Peter Lang, 2015.

CHAPTER SIX

The Uncertainty Relations: Paradigm or Ontology of Nature?

Is it possible to infer which of the elements of Heisenberg's interpretation he held to belong to the basic descriptive ontology of quantum mechanics and which formed merely a part of a proposed paradigm? According to his own account,[1] it was not until some months after the paper on the Uncertainty Relations was written, that he abandoned the belief that the old classical descriptive concepts were inadequate for quantum physics. I surmise then, that in that 1927 paper classical visualizable pictures of quantum phenomena were intended to belong merely to the paradigm and not to the underlying ontology. I conclude then that the underlying descriptive ontology was still controlled by the abstract principle of E (instein)-observability, Heisenberg started out with.

The most important interpretative contribution of this paper is its attempt to explain what is to be understood by the new non-classical quantum mechanical kinematical variables of *place*, *velocity*, *trajectory*, etc. As in the relativistic paradigm, there is a syntactic aspect (of the mathematical model) which escapes sensible intuition and a semantical aspect which reinterprets the variables as constituting an appropriate set of observables. This involves condition Hv of the relativistic model, so far unexploited by Heisenberg. He included reference to an observer (interpreted here as including the measuring instrument) as part of the

1 AHQP, Heisenberg-Kuhn, 27 February 1963.

re-interpreted definition of the variable, and in so far, an epistemological part of the variable as described, as in the case of relativistic space-time. For instance, in his discussion of *place*, Heisenberg writes: "The concept of place necessarily involves reference to a way of measuring position relative to a frame of reference: otherwise the term has no sense."[2]

In the relativistic re-interpretation, the reference was a purely logical one, not taking into account the possible effects of a physical interaction between the object and the observer, an interaction that might possibly affect both. Heisenberg makes clear that in the case of quantum mechanics such an interaction is presumed and enters substantially into what quantum mechanics is all about. A variable is, by definition, an intelligible function of the appropriate measuring process. But, he points out, individual measurements are discrete processes which bind instrument and object through a shared and indivisible photon. In the case of position measurements, these discrete indivisible processes represent no more than a series of discrete locations spaced in time which do not constitute a continuous trajectory. If neighboring locations are joined by straight line segments, neighboring segments have discontinuous slopes on a position-time graph. The discontinuity in slope then measures the velocity (and momentum) uncertainty of the particle.

The new "place" variable is understood as the old intuitively grounded "objectifiable" variable but re-interpreted so as to make it relative to an instrument within the process of a measurement. He then makes the surprising claim: "All the concepts that are used in the classical theory for the description of a mechanical system can also be defined exactly for atomic processes."[3] It is clear from the context, however, that what Heisenberg intends to say is that the classical and quantum mechanical concepts have the same "operational definitions," in other words: the same measuring devices and procedures that are effective in measuring one are also effective in measuring the other. Measurement devices and procedures in the "operational" sense are described in the pre-theoretical language L_P and do not employ the implicitly defined relationships of the theory which would be taken to define a variable in the strict sense of the term. The same pre-theoretically described measurement procedures, he says, can be used to measure classical position and quantum mechanical position.

There is a quantum mechanical limitation, moreover, to the simultaneous observation of canonically conjugate quantities, such as position and momentum: "the experiments which lead to such definitions carry with them an uncertainty

2 Heisenberg (1927c) *op. cit.*, 174.

3 Ibid., 179.

if they involve the simultaneous determination of two canonically conjugate quantities."[4]

Finally, since a quantum mechanical variable is an intelligible function of a measuring process, it is not clear whether the new position variable has observable instances apart from instances that are actually observed—an ambiguity due to Heisenberg's practice of using "observing" and "measuring" as synonymous terms.

Heisenberg tries to explain the simultaneous uncertainty in position and momentum by examples. It is not clear whether the purpose of these examples is to explore the nature of quantum mechanical systems or to provide examples of paradigmatic thinking in quantum mechanics. The latter seems to be the predominant consideration.

The first example concerns an electron of which all that is known is that it would be found on measurement somewhere in the interval (q, q + dq). Heisenberg represents such an electron by a probability amplitude (or wave) S(q), which, by the Born-Pauli statistical rules of interpretation, gives the probability distribution

$$|S(q)|^2\, dq$$

for finding the electron in the position interval (q, q+dq). Heisenberg calls the standard deviation of the distribution Δq, the "position uncertainty of the electron." The probability amplitude S(q) can be converted into the probability amplitude T(p) for the momentum p by the appropriate quantum mechanical transformation rule. T(p) yields the probability distribution

$$|T(p)|^2\, dp$$

for finding the momentum in the interval (p, p + dp). Heisenberg calls the standard deviation of the distribution Δp, the "momentum uncertainty of the electron." Choosing a probability amplitude so as to give a Gaussian wave packet for q (and consequently for p), Heisenberg proves that

$$\Delta q.\ \Delta p > h/2л \qquad (1)$$

This relation he interprets as the "direct intuitive content" of the commutation relation

$$pq - qp = h/2лi$$

4 Ibid.

The mathematical symbols p and q are the matrices (or in Dirac's theory q-numbers) which, according to the rules for "quantizing" a physical problem, replace the classical variables p and q, in the quantum mechanical description.

Δq and Δp, however, are statistical parameters for an ensemble of identically prepared particles and to the extent that intuitive classical notions are called upon, there is no logical reason, as Margenau, Jammer, and others have pointed out, why the commutation relation (1) should impose a limitation on the simultaneous measurability of q and p for an individual particle.[5] The statistical argument just given does not support the conclusion that simultaneous measurability of q and p in individual cases is subject to an Uncertainty Relation. For Heisenberg, however, the Uncertainty Relations state a restriction on the simultaneous measurability of q and p for an individual atomic system. Since this conclusion cannot be derived from the example, it is reasonable to suppose that Heisenberg introduced the example for the purposes of the paradigm alone.

The proof of the Uncertainty Relations for individual cases requires the use of more abstract principles. The proof (only implicit in these papers) follows from an application of the principle of E-observability to the transformation theory outlined in the *Drei Männer Arbeit*.[6] There it is shown that non-commuting matrices cannot be simultaneously diagonalized. Now diagonalizing a matrix displays the set of states in which the physical quantity takes a definite value in a realizable physical environment—that is, it represents the spectrum of observable values of the physical quantity and names the corresponding states of the system. The mathematical fact that non-commuting matrices cannot be simultaneously diagonalized, implies that there is no situation of object-plus-physical environment in which a definite value of one physical quantity co-exists with a definite value of a non-commuting quantity ("non-commuting" referring to the representative matrix operations). Basic to this inference, is the principle of observability that

5 H. Margenau, "Measurements and Quantum States," *Philos. Sci.,* **30** (1963): 1–16, 138–57; M. Jammer, CDQP, 330; P. Feyerabend, "Problems of Microphysics," in *Frontiers of Science and Philosophy*, ed. by R. G. Colodny (London: Allen and Unwin, 1964), 206, 208–17. K. Popper, *The Logic of Scientific Discovery* (London: Hutchinson, 1959), chap. ix. Note that if q and p are not simultaneously measurable, then q and p are not derived from one ensemble of data, but from two ensembles—one from which Δq is derived, and the other from which Δp is derived.

6 A reminder here of E. Husserl's critique of modern science in *The Crisis of European Philosophy and Transcendental Phenomenology,* trans. by D. Carr (Evanston: Northwestern University Press, 1970); due to the loss of philosophical meaning, nature is reduced to a mathematical model.

restricts what can be observed and what, consequently, belongs to the real order within the legitimate interpretation of the mathematical model.[7]

How the Uncertainty Relations affect individual phenomena is dealt with intuitively in a series of examples. For example, Heisenberg considers the limitations imposed by the quantum of action on the ability of an X-ray microscope to localize a particle.[8] The example is worked out in a perfectly classical framework, the atomic system being treated as a classical point particle which interacts during the measurement process with a photon. The example satisfies the need for an intuitive explanation of how and why, within the classical framework, the classical quantities of position and momentum cannot be simultaneously measured. Whether the descriptive variables of the atomic system have (or should be taken to have) a specific quantum theoretic and non-classical meaning is not part of these considerations.

In another example, he treats the diffraction of an electron from a grating.[9] He visualizes the electron as a wave packet occupying a certain volume wider than the spacing of the grating. The spread-out wave packet reflects ignorance of the whereabouts of the electron. More accurate knowledge of the localization of the electron results in a smaller wave packet and less diffraction. Here the paradigm exposition seems to suppose that ignorance of where the electron is positioned within an interval $(q + \Delta q)$ implies a wave function of width Δq and moreover, that diffraction will occur if Δq is larger than the spacing of the diffraction grating.

I now turn to the criticism of this argument. In the first place, while it is true *for a quantum system* that a wave packet of width Δq implies relative ignorance of its position (this implies knowledge merely of the fact that it falls within the interval $(q, q + \Delta q)$), the converse does not follow. Secondly, ignorance of the precise position of a particle within the interval $(q, q + \Delta q)$ does not imply it has a wave function, for it could be a classical particle which is not subject to diffraction.

---

7 For a consideration of the role of models in physics, see E. McMullin "What do Physical Models tell Us?" in *Logic; Methodology and Philosophy of Science III,* ed. by B. van Rootselaar and J. F. Staal (Amsterdam: North-Holland, 1968), 385–96, as well as the references given there to Achinstein, Black, Hesse, and Suppes.

8 Bohr pointed out that in Heisenberg's treatment of the X-ray microscope, there was a serious oversight—he had not taken into account the diameter of the microscope objective lens. Cf. AHQP, Heisenberg-Kuhn, 25 February 1963. Also Jammer CDQM, *op. cit,* 329.

9 Heisenberg distinguishes between the *Schrödinger wave function,* which is a function in 3n-dimensional abstract space (for a system of n particles) and the *wave-packet* in what was soon to be called the "complementary wave picture." The latter was the de Broglie wave-packet visualized in the paradigm as a wave-packet in 3-dimensional classical space, but nevertheless not objectified as an element of the real world.

The argument from ignorance, then, presupposes a great deal that is not explicitly stated: it presupposes that the atomic system is a quantum theoretic object and, therefore, that it possesses a wave function and that its wave function has a known width Δq centered on the expectation value of q. The latter point is an inference derived from the quantum mechanical equation of motion and from the known objective conditions under which the system was prepared. From these, the theoretical solution can be found and an a priori estimate derived of what can be known. A priori limitations on what can be known (an objective uncertainty) need not correspond, however, with the a posteriori limitations on what is actually known (a subjective uncertainty). What Heisenberg meant to affirm is an objective uncertainty, that **is,** a limitation on what can be known. This uncertainty provides an upper limit to what is actually known in any case. It is a theoretical limit that underlies all practical and subjective limits and prepares the ground for a re-definition of the meaning of the term "position," in which the theoretical limit is incorporated in a new meaning as a constitutive element of that meaning. In the new meaning, a continuous trajectory cannot even be defined, since for a continuous trajectory, position and momentum must simultaneously have precise values at every moment. What Heisenberg's Uncertainty Principle shows is that, in the new meaning of the kinematical terms, quantum mechanical systems do not follow continuous trajectories, for the notion of trajectory is not definable in L_Q.

In the course of the paper, the two main themes are reiterated: that the new variables are relative to a measuring environment, and that the relation is based upon a measurement interaction with the environment.[10]

Bohr was critical of the paper in which the Uncertainty Principle was announced. He did not believe that new kinematical concepts were required for quantum physics, and in the course of the following months, he succeeded in bringing Heisenberg around to his view.

10 See Jammer's discussion in CDQM, sect. 7.1.